Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Sample Assessment Material

(Time: 1 hour 30 minutes) | Paper Reference **9FM0/3A**

Further Mathematics

Advanced

Paper 3A: Further Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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1. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\frac{1}{a^2}x^2 - \frac{1}{b^2}y^2 = 1$

differentiate w.r.t. x.

$\frac{2}{a^2}x - \frac{2}{b^2}y\left(\frac{dy}{dx}\right) = 0$

$\frac{2}{a^2}x = \frac{2}{b^2}y\left(\frac{dy}{dx}\right)$

$\frac{dy}{dx} = \frac{2x}{a^2} \div \frac{2y}{b^2} = \frac{2x}{a^2} \times \frac{b^2}{2y} = \frac{xb^2}{ya^2}$

$\left.\frac{dy}{dx}\right|_{(a\sec\theta,\, b\tan\theta)} \quad \frac{a\sec\theta\, b^2}{b\tan\theta\, a^2} = \frac{b\sec\theta}{a\tan\theta}$

sub in coord to find gradient at that point.

$M_{tangent} = b\sec\theta / a\tan\theta$

line eq^n: $y - y_1 = m(x - x_1)$

→ where (x_1, y_1) is known coord. on line.

→ m is gradient

$y - (b\tan\theta) = \frac{b\sec\theta}{a\tan\theta}(x - a\sec\theta)$

$ya\tan\theta - ab\tan^2\theta = xb\sec\theta - ab\sec^2\theta$

$ya\tan\theta - ab\tan^2\theta = xb\sec\theta - ab(1 + \tan^2\theta)$

$ya\tan\theta - \cancel{ab\tan^2\theta} = xb\sec\theta - ab - \cancel{ab\tan^2\theta}$

$ya\tan\theta = xb\sec\theta - ab$ //

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2a. $\Delta OBC = \frac{1}{2}\left|\overrightarrow{OB} \times \overrightarrow{OC}\right|$

$$\overrightarrow{OB} \times \overrightarrow{OC} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

$$\begin{array}{ccc} i & j & k \\ \hline 3 & -1 & 1 \\ 2 & 1 & -1 \end{array} = i\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} - j\begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix} + k\begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix}$$

cross product formulae given in F.B.

$$= i\left[(-1)(-1)-(1)(1)\right] - j\left[(3)(-1)-(1)(2)\right] + k\left[(3)(1)-(2)(-1)\right]$$

$$= 0i + 5j + 5k$$

$$\Delta OBC = \frac{1}{2}\left|\begin{pmatrix} 0 \\ 5 \\ 5 \end{pmatrix}\right|$$

pythagoras

$$= \frac{1}{2}\sqrt{(0)^2+(5)^2+(5)^2}$$

$$= \frac{\sqrt{50}}{2} = \frac{\sqrt{25 \times 2}}{2} = \frac{5\sqrt{2}}{2}$$

$$\Delta OBC = \frac{5\sqrt{2}}{2} \text{ units}^2$$

b. vol. $OABC = \frac{1}{6}\left|\overrightarrow{OA} \cdot \underbrace{(\overrightarrow{OB} \times \overrightarrow{OC})}_{\text{part (a)}}\right|$

$$= \frac{1}{6}\left|\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 5 \\ 5 \end{pmatrix}\right|$$

dot product $= \begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} d \\ e \\ f \end{pmatrix} = (a)(d) + (b)(e) + (c)(f)$

$$= \frac{1}{6}\left|(1)(0)+(1)(5)+(0)(5)\right|$$

$$= \frac{1}{6}|5|$$

$$= \frac{5}{6}$$

vol. $OBCD = \frac{5}{6}$ units

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3a. $t = \tan\left(\frac{x}{2}\right)$

$\sin(x) = \frac{2t}{1+t^2}$

$\cos(x) = \frac{1-t^2}{1+t^2}$

$\tan(x) = \frac{2t}{1-t^2}$

proof @ end of Q

$$4\tan(x) + 3\cot\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right) = 0$$

remember identity $1 + \tan^2(x) = \sec^2(x)$

$$4\tan(x) + \frac{3}{\tan\left(\frac{x}{2}\right)}\left(1 + \tan^2\left(\frac{x}{2}\right)\right) = 0$$

$$4\left(\frac{2t}{1-t^2}\right) + \frac{3}{t}\left(1 + (t)^2\right) = 0$$

$$\frac{8t}{1-t^2} + \frac{3(1+t^2)}{t} = 0$$

$$\frac{8t(t) + 3(1-t^2)(1+t^2)}{t(1-t^2)} = 0$$

$$\frac{8t^2 + 3(1-t^4)}{t(1-t^2)} = 0$$

$$\frac{8t^2 + 3 - 3t^4}{t(1-t^2)} = 0$$

multiply both sides by $t(1-t^2)$

$$-3t^4 + 8t^2 + 3 = 0$$

$\times -1$

$$3t^4 - 8t^2 - 3 = 0 \quad //$$

(shown)

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b. $3t^4 - 8t^2 - 3 = 0$

$(3t^2+1)(t^2-3) = 0$

$t^2 = -1/3$ $t^2 = 3$

$\times$ $t = \pm\sqrt{3}$

gives complex sol's so disregard

remember that $t = \tan\left(\frac{x}{2}\right)$, rearrange and solve for x.

$\frac{x}{2} = \arctan(t)$

$\frac{x}{2} = \arctan(\sqrt{3})$ $-\pi < \frac{x}{2} \leq \pi$

$\frac{x}{2} = \frac{\pi}{3}, -\frac{2\pi}{3}$

$x = \frac{2\pi}{3}, -\frac{4\pi}{3}$

change range to fit $\frac{x}{2}$.

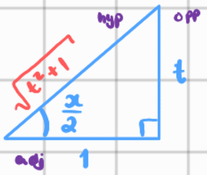
$\frac{x}{2} = \arctan(-\sqrt{3})$ $-\pi < \frac{x}{2} \leq \pi$

$\frac{x}{2} = -\frac{\pi}{3}, \frac{2\pi}{3}$

$x = -\frac{2\pi}{3}, \frac{4\pi}{3}$

$x = -4\pi/3^c, -2\pi/3^c, 2\pi/3^c, 4\pi/3^c$ //

Deriving the t-formulae :



1) draw a right-angled triangle and label angle and sides which you know.

* you are allowed to memorise the t-formulae for $\sin(x)$, $\cos(x)$, $\tan(x)$ and do not have to derive it in the exam unless specifically asked.

2) work out hypotenuse in terms of t, (pythagoras)

$$\sqrt{(t)^2+(1)^2}$$
$$= \sqrt{t^2+1}$$

3) label each side of triangle opposite, adjacent, hypotenuse

4) write out $\sin\left(\frac{x}{2}\right), \cos\left(\frac{x}{2}\right), \tan\left(\frac{x}{2}\right)$ in terms of t.

$$\sin\left(\frac{x}{2}\right) = \frac{t}{\sqrt{t^2+1}} \qquad \sin = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\left(\frac{x}{2}\right) = \frac{1}{\sqrt{t^2+1}} \qquad \cos = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\left(\frac{x}{2}\right) = \frac{t}{1} = t \qquad \tan = \frac{\text{opposite}}{\text{adjacent}} \Big/ \frac{\sin}{\cos}$$

5) Now use double-angle formulae and write in terms of t:

$$\sin(x) = 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)$$
$$\cos(x) = \cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)$$

$$\sin(x) = 2\left(\frac{t}{\sqrt{t^2+1}}\right)\left(\frac{1}{\sqrt{t^2+1}}\right) \qquad \cos(x) = \left(\frac{1}{\sqrt{t^2+1}}\right)^2 - \left(\frac{t}{\sqrt{t^2+1}}\right)^2$$

$$\sin(x) = \frac{2t}{t^2+1} \qquad \cos(x) = \frac{1-t^2}{1+t^2}$$

$$\tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{\frac{2t}{\cancel{1+t^2}}}{\frac{1-t^2}{\cancel{1+t^2}}} = \frac{2t}{1-t^2}$$

$$\tan(x) = \frac{2t}{1-t^2}$$

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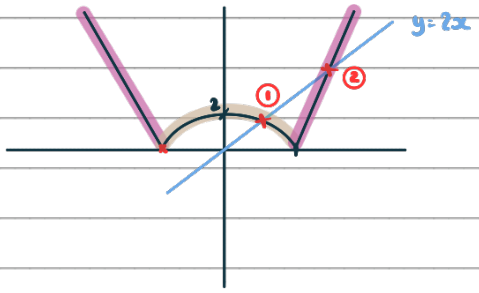
4.



$y = x^2 - 2$

$y = -x^2 + 2$

$|x^2 - 2| > 2x$

Both eq^ns intersect at 2 points.

1 when $y = -x^2 + 2$ and $y = 2x$ intersect

2 when $y = x^2 - 2$ and $y = 2x$ intersect

1// $2x = -x^2 + 2$

$x^2 + 2x - 2 = 0$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - (4)(1)(-2)}}{2(1)} = \frac{-2 \pm \sqrt{12}}{2} = \frac{-1 \pm 2\sqrt{3}}{2} = -1 \pm \sqrt{3}$$

graphically $x \neq -1 - \sqrt{3}$ (-ve. coord).

$\therefore$ 1 $= -1 + \sqrt{3}$

2// $2x = x^2 - 2$

$x^2 - 2x - 2 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - (4)(1)(-2)}}{2(1)} = \frac{2 \pm \sqrt{12}}{2} = \frac{1 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$$

graphically $x \neq 1 - \sqrt{3}$ (-ve. coord).

$\therefore$ 2 $= 1 + \sqrt{3}$

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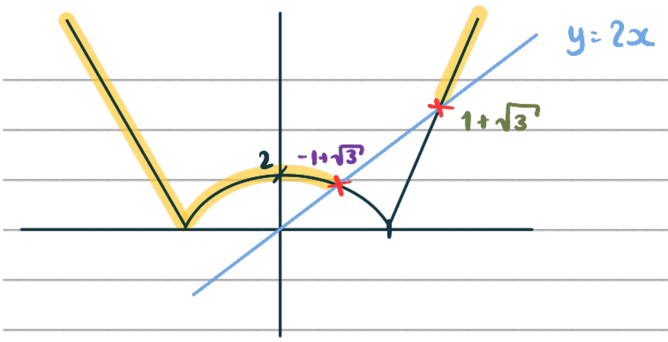

← represents when

$|x^2-2| > 2x$

$\{x \in \mathbb{R} : x < -1+\sqrt{3}\} \cup \{x \in \mathbb{R} : x > 1+\sqrt{3}\}$

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5 i. $a \cdot (b \times a)$ remember cyclic property

$b \cdot (a \times a)$

$b \cdot (0)$ 2 vector product property (if 2 vectors are parallel cross product of both = 0)

$= 0$

$\therefore\ a \cdot (b \times a) = 0$ //

$a \cdot (b \times a) = 0$

move each constant left

$b \cdot (a \times a) = 0$

move each constant left

$a \cdot (a \times b) = 0$

all mean same thing

ii. $a \times b = a \times c$

$(a \times b) - (a \times c) = 0$ move to same side & factorise

$a \times (b - c) = 0$

since $a \neq 0$ and $b - c \neq 0$, a must be parallel in order for

$a \times (b - c) = 0$

$\therefore\ b - c = \lambda a$ // (shown)

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6 a. $x_0 = 0$ $y_0 = 1$

$$\left(\frac{dy}{dx}\right)_0 \quad (0)^2 - (1)^2 = -1$$

$h = 0.05$ and need to find y when $x = 0.1$

$\therefore \frac{0.1 - 0}{0.05} = 2$ need 2 iterations (find y_2)

$$y_1 \approx y_0 + h\left(\frac{dy}{dx}\right)_0$$
$$y_1 \approx 1 + (0.05)(-1)$$
$$y_1 \approx 0.95$$

$x_1 = 0.05$ $y_1 = 0.95$

$$\left(\frac{dy}{dx}\right)_1 = (0.05)^2 - (0.95)^2$$
$$= -0.9$$

$$y_2 \approx y_1 + h\left(\frac{dy}{dx}\right)_1$$
$$y_2 \approx 0.95 + (0.05)(-0.9)$$
$$y_2 \approx 0.905$$

$y = 0.905$ //

b. $\frac{dy}{dx} = x^2 - y^2$

differentiate w.r.t. x

$$\frac{d^2y}{dx^2} = 2x - 2y\left(\frac{dy}{dx}\right)$$

differentiate w.r.t. x

$$\frac{d^3y}{dx^3} = 2 - 2\left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right) - 2y\left(\frac{d^2y}{dx^2}\right)$$
$$= 2 - 2\left(\frac{dy}{dx}\right)^2 - 2y\left(\frac{d^2y}{dx^2}\right)$$

$$\frac{d^3y}{dx^3} = 2 - 2\left(\frac{dy}{dx}\right)^2 - 2y\left(\frac{d^2y}{dx^2}\right) \text{ //}$$

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c. $x_0 = 0 \quad y_0 = 1$

$$\left.\frac{dy}{dx}\right|_{(0,1)} \quad (0)^2 - (1)^2 = -1$$

$$\left.\frac{d^2y}{dx^2}\right|_{(0,1)} \quad 2(0) - 2(1)(-1) = 2$$

$$\left.\frac{d^3y}{dx^3}\right|_{(0,1)} \quad 2 - 2(-1)^2 - 2(1)(2) = -4$$

$$\boxed{y = y_0 + x\left.\frac{dy}{dx}\right|_{x=0} + \frac{x^2}{2!}\left.\frac{d^2y}{dx^2}\right|_{x=0} + \frac{x^3}{3!}\left.\frac{d^3y}{dx^3}\right|_{x=0} + \dots}$$

$$y = 1 + x(-1) + \frac{x^2}{2}(2) + \frac{x^3}{6}(-4)$$

$$y = 1 - x + x^2 - \frac{2}{3}x^3 + \dots$$

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7a. $A = 1000\left(1 + \frac{5}{100(12)}\right)^{12}$

$A = 100$
$r = 5$
$n = 12$

$A = 1051.161898$

$A = £1051.16 \quad (2 \text{ d.p.})$ // (shown)

b. let $y = \left(1 + \frac{r}{100n}\right)^n$

$\ln(y) = \ln\left(1 + \frac{r}{100n}\right)^n = n \ln\left(1 + \frac{r}{100n}\right)$

$\lim_{n\to\infty} \ln(y) = \lim_{n\to\infty} n\ln\left(1 + \frac{r}{100n}\right)$

$= \lim_{n\to\infty} \frac{\ln\left(1 + \frac{r}{100n}\right)}{\frac{1}{n}}$

is now in a form where you can apply L'Hopital's Rule

L'Hopital's Rule states if $\lim_{n\to a} \frac{f(n)}{g(n)}$ then $\lim_{n\to a} \frac{f'(n)}{g'(n)}$

let $f(n) = \ln\left(1 + \frac{r}{100n}\right) = \ln\left(1 + \frac{1}{100} r n^{-1}\right)$

$f'(n) = \frac{\frac{-rn^{-2}}{100}}{1 + \frac{r}{100n}} = \frac{\frac{-r}{100n^2}}{1 + \frac{r}{100n}}$

let $g(n) = \frac{1}{n} = n^{-1}$

$g'(n) = -n^{-2} = -1/n^2$

(-ve signs cancel out)

$\lim_{n\to\infty}\left(\frac{\frac{-r}{100n^2}}{1 + \frac{r}{100n}} \div -\frac{1}{n^2}\right) = \lim_{n\to\infty}\left(\frac{r}{100\cancel{n^2}\left(1 + \frac{r}{100n}\right)} \times \cancel{n^2}\right) = \lim_{n\to\infty}\left(\frac{\frac{r}{100}}{1 + \frac{r}{100n}}\right)$

$\lim_{n\to\infty}\left(\frac{\frac{r}{100}}{1 + \frac{r}{100n}}\right) = \frac{r}{100}$ (as $n \to \infty$ $1 + \frac{r}{100n} \to 1$, $\left[\frac{r}{100n} \to 0\right]$ leaves $r/100$ remaining)

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$\ln(y) = \frac{r}{100}$ ← equating from earlier

$y = e^{\frac{r}{100}}$ // solve in terms of y

c. using $r = 5$,

$\lim_{n \to \infty} \left(1 + \frac{5}{100n}\right)^n = e^{0.05}$

$\therefore \lim_{n \to \infty} 1000\left(1 + \frac{5}{100n}\right)^n = 1000e^{0.05}$

$= £1051.271096$

$\approx £1051.27$ saved after 1 year //

8a. $x = wt$

$\frac{dx}{dt} = w + t\frac{dw}{dt}$ } differentiate w.r.t. t.

$t\left(w + t\,{}^{dw}/_{dt}\right) + 2t^2(wt) = (wt)(2t+1)$

$tw + t^2\,{}^{dw}/_{dt} + 2t^3w = 2t^2w + tw$

$t^2\,{}^{dw}/_{dt} + 2t^3w = 2t^2w$

$\frac{dw}{dt} + 2tw = 2w$

sub in x and $\frac{dx}{dt}$ worked out from above to create new eqⁿ in terms of w and t.

$\frac{dw}{dt} + 2tw - 2w = 0$ //

(Shown)

b. $\frac{dw}{dt} = 2w - 2tw$

$\frac{dw}{dt} = 2w(1-t)$

$\frac{1}{w}\,dw = 2(1-t)\,dt$

Seperate variables

$\int \frac{1}{w}\,dw = \int 2(1-t)\,dt$

integrate both sides to eliminate dw and dt terms

$\ln(w) = \int 2 - 2t\;dt$

$\ln(w) = 2t - t^2 \quad c$

get rid of ln

$w = e^{2t - t^2 + c}$

$x^{m+n} = x^m \times x^n$ (index law)

$w = \left(e^{2t-t^2}\right) \times e^c$

let $A = e^c$ as e^c is a constant

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$\therefore\ w = Ae^{2t-t^2}$

convert back in terms of x and t.

$\frac{x}{t} = Ae^{2t-t^2}$

$x = Ate^{2t-t^2}$ // (shown)

c. when $t=2$, $x=10$

$10 = A(2)e^{2(2)-(2)^2}$

$10 = 2Ae^0 = 2A(1)$

$10 = 2A$

$A = 5$

$x = 5te^{2t-t^2}$ //

d. @ max. displacement, $\frac{dx}{dt} = 0$

$x = \underbrace{5t}_{\alpha}\underbrace{e^{2t-t^2}}_{\beta}$

$\frac{dx}{dt} = 5e^{2t-t^2} + 5t(2-2t)e^{2t-t^2}$

$5e^{2t-t^2} + 5t(2-2t)e^{2t-t^2} = 0$ $\quad \div e^{2t-t^2}$

$5 + 5t(2-2t) = 0$

$5 + 10t - 10t^2 = 0$

$10t^2 - 10t - 5 = 0$ $\quad \div 5$

$2t^2 - 2t - 1 = 0$

solve for t via quadratic formulae

$t = \frac{-(-2) \pm \sqrt{(-2)^2 - (4 \times 2 \times -1)}}{2(2)} = \frac{1 \pm \sqrt{3}}{2}$

remember product rule & chain rule:

- $\frac{d}{dx}(\alpha\beta) = \alpha\beta' + \alpha'\beta$
- $\frac{d}{dx}(f(g(x))) = f'(g(x))g(x)$

$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)e^{f(x)}$

Let $f(x) = 2t - t^2$

$f'(x) = 2 - 2t$

$\frac{d}{dx}\left(e^{2t-t^2}\right) = (2-2t)e^{2t-t^2}$

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$t > 0 \quad \therefore t \neq \frac{1-\sqrt{3}}{2}$

$\therefore t = \frac{1+\sqrt{3}}{2}$

To find max. displacement, sub in t-value into x eqn

$x = 5\left(\frac{1+\sqrt{3}}{2}\right)e^{2\left(\frac{1+\sqrt{3}}{2}\right) - \left(\frac{1+\sqrt{3}}{2}\right)^2}$

$x = 16.23823545$

max. displacement = 16.2 cm //

e. as $t \to \infty$, $e^{2t - t^2} \to 0$

$\therefore 5te^{2t-t^2} \to 0$

$\therefore$ as $t \to \infty$, displacement tends to 0 //

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9a. $|AD| = \frac{16}{3}\sqrt{3}$

$\frac{16}{3}\sqrt{3} \div 2 = \frac{8\sqrt{3}}{3}$) distance from y-axis to point A or D.

$D\left(\frac{8\sqrt{3}}{3}, y\right)$

$A\left(-\frac{8\sqrt{3}}{3}, y\right)$

Since AB and CD are directrices: $x = \pm\frac{a}{e}$

we know $x = \pm\frac{8\sqrt{3}}{3}$ are directrices

$\frac{a}{e} = \frac{8\sqrt{3}}{3}$

We know $a = 2b$

use eccentricity formulae

$b^2 = a^2(1-e^2)$

$b^2 = (2b)^2(1-e^2)$

$b^2 = 4b^2(1-e^2)$

$\frac{b^2}{4b^2} = 1-e^2$

Conics

	Ellipse	Parabola	Hyperbola	Rectangular Hyperbola
Standard Form	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$y^2=4ax$	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$xy=c^2$
Parametric Form	$(a\cos\theta, b\sin\theta)$	$(at^2, 2at)$	$(a\sec\theta, b\tan\theta)$ $(\pm a\cosh\theta, b\sinh\theta)$	$\left(ct, \frac{c}{t}\right)$
Eccentricity	$e<1$ $b^2=a^2(1-e^2)$	$e=1$	$e>1$ $b^2=a^2(e^2-1)$	$e=\sqrt{2}$
Foci	$(\pm ae, 0)$	$(a, 0)$	$(\pm ae, 0)$	$(\pm\sqrt{2}c, \pm\sqrt{2}c)$
Directrices	$x=\pm\frac{a}{e}$	$x=-a$	$x=\pm\frac{a}{e}$	$x+y=\pm\sqrt{2}c$
Asymptotes	none	none	$\frac{x}{a}=\pm\frac{y}{b}$	$x=0, y=0$

$\frac{1}{4} = 1-e^2$

$e^2 = 3/4$

$e = \pm\sqrt{\frac{3}{4}} = \pm\sqrt{3}/2$

$0<e<1$

$\therefore e = +\sqrt{3}/2$

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$x = \frac{a}{e} = \frac{8\sqrt{3}}{3}$

$\frac{a}{e} = \frac{8\sqrt{3}}{3}$

$\frac{a}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{8\sqrt{3}}{3}$

Sub in e value calculated and work out a.

$a = \frac{8\sqrt{3}}{3} \times \frac{\sqrt{3}}{2} = \frac{8(3)}{6} = \frac{24}{6} = 4$

$a = 4 \Rightarrow a^2 = 16$

$a = 2b \quad 4 = 2b \Rightarrow b = 2 \Rightarrow b^2 = 4$

$\frac{x^2}{16} + \frac{y^2}{4} = 1$ // ← in form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

b. calculate foci of ellipse :

foci : $(\pm ae, 0)$

$= \left(\pm (4)\left(\frac{\sqrt{3}}{2}\right), 0\right)$

$= (\pm 2\sqrt{3}, 0)$

$(2\sqrt{3}, 0)$ and $(-2\sqrt{3}, 0)$ //

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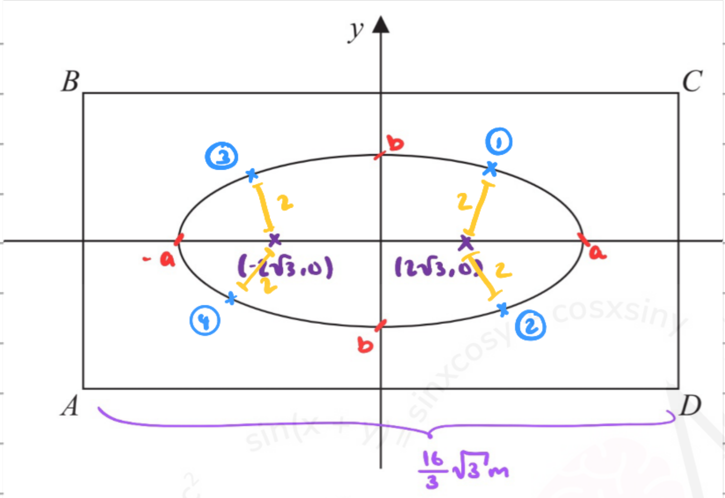
c. Due to symmetry of Ellipse, 4 possible coordinates we are trying to calculate



Let ① be written in parametric form, $(a\cos\theta, b\sin\theta)$

$= (4\cos\theta, 2\sin\theta)$

distance from $(4\cos\theta, 2\sin\theta)$ to $(2\sqrt{3}, 0)$ is 2

$$\sqrt{(4\cos\theta - 2\sqrt{3})^2 + (2\sin\theta)^2} = 2$$
$$(4\cos\theta - 2\sqrt{3})^2 + (2\sin\theta)^2 = 4$$
$$16\cos^2\theta - 16\sqrt{3}\cos\theta + 12 + 4\sin^2\theta = 4$$
$$4\cos^2\theta + 4\sin^2\theta + 12\cos^2\theta - 16\sqrt{3}\cos\theta + 8 = 0$$
$$4\underbrace{[\cos^2\theta + \sin^2\theta]}_{=1} + 12\cos^2\theta - 16\sqrt{3}\cos\theta + 8 = 0$$

$$4 + 12\cos^2\theta - 16\sqrt{3}\cos\theta + 8 = 0$$
$$12\cos^2\theta - 16\sqrt{3}\cos\theta + 12 = 0 \quad \div 4$$
$$3\cos^2\theta - 4\sqrt{3}\cos\theta + 3 = 0$$
$$(3\cos\theta - \sqrt{3})(\cos\theta - \sqrt{3}) = 0$$

$\cos\theta = \frac{\sqrt{3}}{3}$ or $\cos\theta = \sqrt{3}$

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x-coord : $4\cos\theta$

if $\cos\theta = \frac{\sqrt{3}}{3}$ $\quad x = 4\left(\frac{\sqrt{3}}{3}\right) = \frac{4\sqrt{3}}{3}$

if $\cos\theta = \sqrt{3}$ $\quad x = 4\left(\sqrt{3}\right) = 4\sqrt{3}$ (this is not on ellipse so $\cos\theta \neq \sqrt{3}$)

Since the gnomes are symmetrical, @ ③ and ④ x-coord $= \left(-\frac{4}{3}\sqrt{3}\right)$.

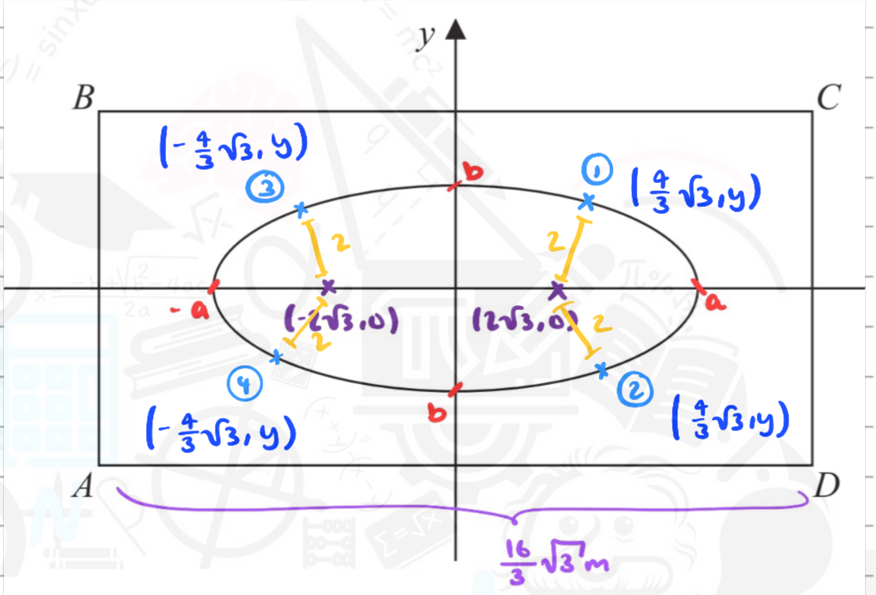
Since $\cos\theta = \frac{\sqrt{3}}{3}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(\sqrt{3}/3\right)^2 = 1$

$\sin^2\theta = 1 - \left(\sqrt{3}/3\right)^2 = 2/3$

$\sin\theta = \pm\sqrt{\frac{2}{3}} = \pm\frac{\sqrt{6}}{3}$



y-coord : $2\sin\theta$

if $\sin\theta = +\frac{\sqrt{6}}{3}$ $\quad y = 2\left(\frac{\sqrt{6}}{3}\right) = \frac{2\sqrt{6}}{3}$

if $\cos\theta = -\frac{\sqrt{6}}{3}$ $\quad y = 2\left(-\frac{\sqrt{6}}{3}\right) = -\frac{2\sqrt{6}}{3}$

} can be either

Coord. ① : $\left(\frac{4\sqrt{3}}{3}, \frac{2\sqrt{6}}{3}\right)$

Coord. ② : $\left(-\frac{4\sqrt{3}}{3}, \frac{2\sqrt{6}}{3}\right)$

Coord. ③ : $\left(\frac{4\sqrt{3}}{3}, -\frac{2\sqrt{6}}{3}\right)$

Coord. ④ : $\left(-\frac{4\sqrt{3}}{3}, -\frac{2\sqrt{6}}{3}\right)$